Recent Results on Integrable and Non-integrable Lotka Volterra Systems

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Talk presented at the 3rd International Conference on Integrable Systems and Nonlinear Dynamics Yaroslavl, Russia, 4-8 October 2021

In recent years, there has been renewed interest in the study of anti-symmetric Lotka Volterra Hamiltonian (LVH) systems of competing species, $x_i(t)$, satisfying the ODEs

$$dx_i/dt = \sum_{i=1}^{n} a_{ij} x_i x_j = h = const., a_{i,j} = -a_{j,i},\ \ i,j = 1,2,...,n$$

and preserving the sum $\sum_{i=1}^{n} x_i(t) = h = const.$ [1]. In particular, it is interesting to add linear (or nonlinear) terms to these systems, and either seek to preserve integrability, or investigate the dynamics of "nearby" nonintegrable systems in the $n-$ dimensional phase space [2]. In this talk, I will first show how new integrable classes of LVH systems were discovered applying the Painlevé property [3], and then demonstrate that "nearby" non-integrable systems typically continue to possess very simple dynamics. Finally, I will discuss some very recent results revealing interesting comparisons between the Painlevé property and Brenig's method of integrating polynomial systems of ODEs by reduction to canonical form [4, 5].

References

[1] Bountis T. Vanhaecke P., *Lotka – Volterra Systems Satisfying a Strong Painlevé Property*, Physics Letters A, 380(47), 9 December 2016, Pages 3977–3982. See also: PLA 381(45), 6 December 2017, Page 3843.

[2] Bountis T., Zhunussova Zh., Dosmagulova K., Kanellopoulos G., *Integrable and Non-integrable Lotka Volterra Systems*, Physics Letters A, Vol. 402, 127360 (June 28, 2021).

[3] Ramani A., Grammaticos B. and Bountis T., *The Painlevé Property and Singularity Analysis of Integrable and Non-Integrable Systems*, Physics Reports, 180 (3), 159 (1989).

[4] Brenig L., *Reducing Dynamical Systems to Canonical Forms*, Phil. Trans. A, Proc. Royal Soc. 376, 20170384 (2018).

[5] Bountis T., Brenig L., *Connections Between the QP Formalism and the Painlevé Property in Integrable Dynamical Systems*, submitted for publication (2021).