Diffusion without spreading of a Wavepacket in Nonlinear Random Lattices [4]

Serge AUBRY
Laboratoire Léon Brillouin (CEA-CNRS) CEA Saclay, 91191-Gif-sur-Yvette cedex, France

An interesting question is: What is the behavior of a wavepacket in a model with linear Anderson localization when extra nonlinearities with higher order are taken into account? Numerical simulations [1] have shown in such cases by averaging over many disorder realizations that a wave packet exhibits a subdiffusive behavior with exponent about 1/3.

However, it was proved in some models on infinite lattice and later numerically observed on very large systems that small amplitude wavepackets could generate stationary quasiperiodic solutions (KAM tori) with a probability which goes to 1 as their amplitude go to zero. Then we concluded [1] that any initially localized wave packet could not spread to zero because either it generates such a quasiperiodic solution if its amplitude is small enough or if it is initially chaotic, it cannot spread to zero because of the forbidden region of KAM tori at small amplitude.

Assuming that such chaotic trajectories fill ergodically the available phase space region complementary to the KAM tori (Arnold conjecture), we conclude that such chaotic wavepackets though they must remain focused around one or few finite amplitude peaks, should be randomly moving through the whole system. Note that this feature has been recently observed [2].

We also investigate a special class of models on arbitrary lattices at any dimension where the nonlinearities are replaced by hardcore potentials (Ding-Dong models). Numerical investigations [3] in 1 dimension have shown they also exhibit subdiffusive behavior with exponent about 1/3 similarly to general models. In that class of models, we prove rigorously the absence of wavepacket spreading in all cases.

In summary, our conclusion is that the Anderson localization of any wavepacket is not destroyed by nonlinearity. Either the initial wave packet (mostly at small amplitude), remains a quasiperiodic stationnary solution (which is nearly the same as those obtained in the purely linear case), or at larger amplitude, it remains localized around one of few chaotic peaks though these peaks move randomly. We suggest that subdiffusion could be understood by assuming these peaks move more or less like a random walk in a random scenery.

[1] S. Aubry, International Journal of Bifurcation and Chaos, **21** 2125–2145 (2011) and references therein
[2] C. Skokos, E. Gerlach, S. Flach, ArXiv2112.04190v1 (2021)
[3] A. Pikovsky, J. Stat. Mech. (2020) 053301
[4] S. Aubry, in preparation