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Fokas on medical imaging: Analytic reconstructions for emission tomography

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Mathematical problems associated with the theoretical foundations of emission tomography involve the inversion of the celebrated Radon transform of a function, defined as the set of all its line integrals, as well as the inversion of a certain generalization of the Radon transform of a function, the so-called *attenuated Radon transform*, defined as the set of all its *attenuated* line integrals [1]. The non-attenuated and attenuated versions of the Radon transform provide the mathematical basis of emission tomography, particularly of two of the most important available medical imaging techniques, namely positron emission tomography (PET) [2], and single-photon emission computed tomography (SPECT) [3]. Although Radon himself derived in 1917 the inversion of the transform bearing his name, seventy four years later Novikov and Fokas rederived this well-known formula by considering two classical problems in complex analysis known as the $\bar{d}$-problem and the scalar Riemann-Hilbert problem. The inversion may be obtained in a simpler manner by the use of the Fourier transform, however the derivation of Novikov and Fokas allowed Novikov to invert the attenuated Radon transform in 2002. It took Fokas, Iserles and Marinakis four more years to establish a more straightforward derivation of this inversion. In this lecture, we shall present the seminal work of Fokas in the area of mathematical image reconstruction, based on the mathematical machinery of modern methods in complex analysis.

References

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