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Chaos in Hamiltonian systems

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In this talk we will present the concept of chaos in the framework of the Hamiltonian formalism used to describe the evolution of dynamical systems. Among the three conditions required for characterizing a dynamical system as chaotic, i.e. topological transitivity, the existence of a dense set of periodic orbits and the sensitive dependence on initial conditions [1] we will mainly focus on the latter.

Using as our main example, the prototypical model of the two degree of freedom Hénon – Heiles system [2], which tries to describe the motion of a star at the central parts of a galaxy, we will present various numerical manifestations of chaos. For example, we will use the so-called Poincaré surface of section method (see e.g. [3, pp. 17–20]) to visualize regions of chaotic and regular behavior, and the maximum Lyapunov exponent (MLE) [4, 5] to quantify chaoticity.

In addition, we will discuss two very efficient methods of chaos detection: the Smaller (SALI) and the Generalized (GALI) Alignment Index techniques (see [6] and references therein). In particular, we will first recall the definitions of the SALI and the GALI and then discuss the behavior of these indices for regular and chaotic orbits. We will also show that the SALI/GALI techniques are reliable chaos indicators, which proved to be much more efficient than the computation of the MLE.

References

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