28th SUMMER SCHOOL-CONFERENCE ON DYNAMICAL SYSTEMS
AND COMPLEXITY, CHANIA, CRETE, 18 - 26 JULY 2022

On the Shape and Size of Granular Roll Waves

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Granular roll waves are steady travelling wave solutions of the generalized Saint-Venant equations for flowing granular matter [1-6]. They appear when the Froude number of the base flow exceeds a critical value, $Fr > Fr_{cr}$, and are in fact the only stable waveform in this regime. In the phase space of the corresponding dynamical system, the roll waves take the form of a stable limit cycle around an unstable fixed point.

In this talk [1] we demonstrate how the stable and unstable manifolds of a neighbouring saddle point, in combination with the nullclines, determine the precise shape of the roll waves. We find that, for a given Froude number, the limit cycle cannot become arbitrarily large; the upper bound is related to the homoclinic bifurcation occurring when the stable and unstable manifolds smoothly connect to form a homoclinic loop.

Forcing the system past this bifurcation, an opening is created between these manifolds through which the trajectories in phase space escape. We show that this escape signals a spontaneous readjustment of the roll waves to match the new, post-bifurcation conditions.

References

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