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A Dynamical Systems View of Granular Waves

KO VAN DER WEELE, DIMITRIOS RAZIS & GIORGOS KANELLOPOULOS

Department of Mathematics
University of Patras, Patras 26500 Greece
weele@math.upatras.gr • https://www.math.upatras.gr/en/people/weele

Granular flows on inclined planes are ubiquitous in agriculture and industrial applications (wherever grains, sand, or mineral ores are transported) as well as in the natural environment, where such flows can take the form of devastating landslides and rock avalanches. In this talk we present a Dynamical Systems approach to explore the various types of travelling surface waves that are encountered in this kind of flow [1-6].

For small values of the Froude number Fr, the granular material will slide downward either as a sheet of constant thickness ("uniform flow") or in the form of a "monoclinal flood wave", consisting of two uniform flows connected by a shock structure. By contrast, when Fr exceeds a critical value, the flow organizes itself in a periodic train of "roll waves".

On the basis of the generalized Saint-Venant equations for granular flow, we derive a dynamical system that captures these – and many other – waveforms as specific bounded orbits in phase space. Focusing upon the transition from monoclinal flood waves to roll waves, by gradually increasing Fr through its critical value, we find that this transition features a whole spectrum of intermediate stages (including an "undular bore" and a special type of solitary wave) that had hitherto not been reported for granular flows.

References

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