28th SUMMER SCHOOL-CONFERENCE ON DYNAMICAL SYSTEMS
AND COMPLEXITY, CHANIA, CRETE, 18 - 26 JULY 2022

Brief Introduction to Quantum Chaos

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I shall review the basic aspects of quantum chaos (wave chaos) [1,2] in general (mixed-type) Hamiltonian systems with divided phase space, where regular regions containing the invariant tori coexist with the chaotic regions [3,4]. The quantum evolution of classically chaotic bound systems does not possess the sensitive dependence on initial conditions, and thus no chaotic behaviour occurs, as the motion is always almost periodic. However, the study of the stationary solutions of the Schrödinger equation in the quantum phase space (Wigner functions or Husimi functions [5,6]) reveals precise analogy of the structure of the classical phase portrait. In classically integrable regions the spectral (energy) statistics is Poissonian, while in the ergodic chaotic regions the random matrix theory applies. One important indicator of the level statistics is the probability density (level spacing distribution) $P(S)$ to find successive levels on a distance S. In the integrable case the level spacing distribution is exponential $P(S) = \exp(-S)$, while in the chaotic case it is well described by the Wigner distribution

$$P(S) = \frac{\pi S}{2} \exp(-\frac{\pi S^2}{4}). \tag{1}$$

If we have the mixed-type classical phase space, in the semiclassical limit (short wavelength approximation) the spectrum is composed of Poissonian level sequence supported by the regular part of the phase space, and chaotic sequences supported by classically chaotic regions, being statistically independent of each other, as described by the Berry-Robnik distribution [7-11]. In quantum systems with discrete energy spectrum the Heisenberg time $t_H = 2\pi\hbar/\Delta E$, where ΔE is the mean level spacing (inverse energy level density), is an important time scale. The classical transport time scale t_T (transport time) in relation to the Heisenberg time scale t_H (their ratio is the parameter $\alpha = t_H/t_T$) determines the degree of localization of the chaotic eigenstates[12-19], whose measure A is based on the information entropy. We show that A is linearly related to the normalized inverse participation ratio. We study the structure of quantum localized chaotic eigenstates (their Wigner and Husimi functions) and the distribution of localization measure A. The latter one is well described by the beta distribution, if there are no sticky regions in the classical phase space. Otherwise, they have a complex nonuniversal structure. We show that the localized chaotic states display the fractional power-law repulsion between the nearest energy levels where $P(S)$ goes like $\propto S^\beta$ for small S, where $0 \leq \beta \leq 1$, and $\beta = 1$

corresponds to completely extended states, while $\beta = 0$ to the maximally localized states. β goes from 0 to 1 when α goes from 0 to ∞. β is a function of $\langle A \rangle$, as demonstrated in the quantum kicked rotator, the stadium billiard, a mixed-type billiard and in the Dicke model [20,21].

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