28th SUMMER SCHOOL-CONFERENCE ON DYNAMICAL SYSTEMS
AND COMPLEXITY, CHANIA, CRETE, 18 - 26 JULY 2022

Progress in initial-boundary value problems for nonlinear evolution equations and the Fokas method

ALEX HIMONAS

Department of Mathematics
University of Notre Dame, Notre Dame, IN 46556, USA
himonas.1@nd.edu • https://math.nd.edu/people/faculty/alex-himonas/

The unified transform method (UTM), also known as the Fokas method, provides a novel approach for solving initial-boundary value problems (ibvp's) for linear and integrable nonlinear partial differential equations. In particular, it gives solution formulas for forced linear ibvp's. This motivated the initiation of a new program (by Fokas and collaborators) for studying the well-posedness in Sobolev spaces of ibvp's for nonlinear evolution equations by employing this method, which is analogous to the way well-posedness of initial value problems (ivp's) are studied. Using the Fokas formula we are able to derive linear estimates in Sobolev, Hadamard and Bourgain spaces. Then, using as an iteration map the one defined by the UTM formula when the forcing is replaced by the nonlinearity, and utilizing the linear estimates in combination with the multilinear estimates suggested by the nonlinearity we show that this map is a contraction in appropriate solution spaces. For the Korteweg-de Vries and Nonlinear Schrödinger equations this program has been implemented for ibvp in one-space dimension [1,2,4] and significant progress has been made in higher dimensions [3]. In this talk we will try to present key points of this remarkable story. The talk is based on collaborative work with Thanasis Fokas, Dionyssis Mantzavinos and Fangchi Yan.

References

[1] A. Fokas, A. Himonas and D. Mantzavinos, "The Korteweg-de Vries equation on the half-line", Nonlinearity **29**, 489-527 (2016).

[2] A. Fokas, A. Himonas and D. Mantzavinos, "The nonlinear Schrödinger equation on the half-line", Trans. Amer. Math. Soc. **3699**, 681-709 (2017).

[3] A. Himonas and D. Mantzavinos, "Well-posedness of the nonlinear Schrödinger equation on the half-plane", Nonlinearity **33**, 5567-5609 (2020).

[4] A. Himonas and F. Yan, "The Korteweg-de Vries equation on the half-line with Robin and Neumann data in low regularity spaces", Nonlinear Anal. **222**, 113008 (2022).