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Hydraulic Jumps: A New Perspective through Dynamical Systems

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Hydraulic jumps mark the sudden transition from a fast, relatively thin supercritical flow to a slower and thicker subcritical one. Here we focus on “weak jumps” in laminar open channel flow, which we model by means of the generalized Saint-Venant equations, expressing the mass and momentum balances [1-3].

The hydraulic jumps in question arise as stable stationary solutions of these equations. They prove to be ideally suited for a Dynamical Systems analysis: in phase space they manifest themselves as marked near-parabolic trajectories, following the unstable manifold of the system’s sole fixed point which, importantly, is a *saddle* [1,2]. It is the hybrid attracting/repelling nature of this saddle that is responsible for the jump phenomenon: the trajectory is first attracted towards the saddle but, upon reaching its vicinity, suddenly gets catapulted away from it along the aforementioned near-parabolic orbit.

Based on this geometric interpretation, we derive an analytic expression for the jump length in terms of Fr and Re (the Froude and effective Reynolds number, respectively), reflecting the fact that gravity and viscous diffusion both contribute to the balance of forces that shape these laminar hydraulic jumps [1].

References

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