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Robust and verified Koopmania!

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Koopman operators are infinite-dimensional operators that globally linearise non-linear dynamical systems, making their spectral information valuable for understanding dynamics. Their growing popularity, dubbed "Koopmania", has produced 10,000s of articles over the past decade. However, Koopman operators can have continuous spectra, can lack finite-dimensional invariant subspaces, and approximations can suffer from spectral pollution (spurious modes). These issues make computing the spectral properties of Koopman operators a considerable challenge. In this talk, we present data-driven algorithms with rigorous convergence guarantees for computing spectral properties of Koopman operators from trajectory data [1,2]. We present the first algorithm for computing the spectra and pseudospectra of general Koopman operators from trajectory data without spectral pollution, namely residual dynamic mode decomposition (ResDMD). By combining ResDMD and the resolvent, we compute smoothed approximations of spectral measures associated with measure-preserving dynamical systems. When computing the continuous and discrete spectrum, explicit convergence theorems provide high-order convergence, even for chaotic systems. Kernelized variants of our algorithms allow for dynamical systems with a high-dimensional state-space, and the error control provided by ResDMD allows a posteriori verification of learnt dictionaries. For example, we compute spectral measures for a protein molecule (20,046-dimensional state-space) and compute nonlinear Koopman modes with error bounds for chaotic turbulent flow past aerofoils (295,122-dimensional state-space, Reynolds number $> 100,000$).

References

[1] M.J. Colbrook, A. Townsend, "Rigorous data-driven computation of spectral properties of Koopman operators for dynamical systems", arXiv:2111.14889 (2021).

[2] M.J. Colbrook, L. Ayton, M. Szőke, "Residual Dynamic Mode Decomposition: Robust and verified Koopmanism", `http://www.damtp.cam.ac.uk/user/mjc249/pdfs/ResDMD.pdf` (2022).